# RESTRICTED 內部文件

香　港　考　試　局
HONG KONG EXAMINATIONS AUTHORITY

一　九　八　八　年　香　港　中　學　會　考
HONG KONG CERTIFICATE OF EDUCATION EXAMINATION, 1988

## 數學
## Mathematics

## 評卷參考
## Marking Scheme



**請在學校任教之閱卷員特別留意**

本評卷參考並非標準答案，故極不宜落於學生手中，以免引起誤會。

遇有學生求取此文件時，閱卷員應嚴予拒絕。閱卷員在任何情況下披露本評卷參考內容，均有違閱卷員守則及「一九七七年香港考試局法例」。

**Special Notes for Teacher Markers**

It is highly undesirable that this marking scheme should fall into the hands of students. They are likely to regard it as a set of model answers, which it certainly is not.

Markers should therefore resist pleas from their students to have access to this document. Making it available would constitute mis-conduct on the part of the marker and is, moreover in breach of the 1977 Hong Kong Examinations Authority Ordinance.





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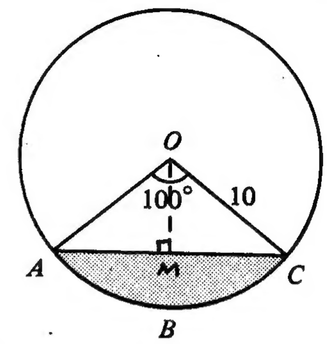
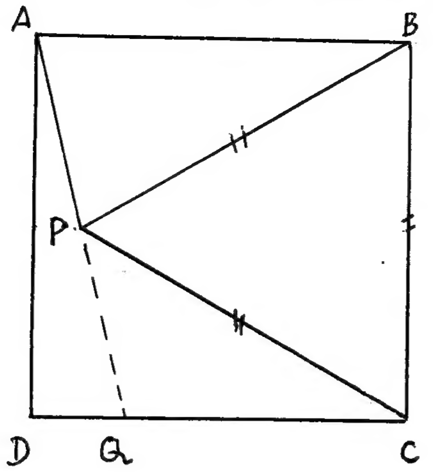
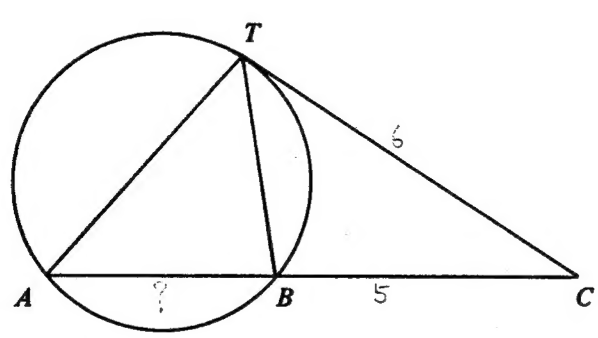
| | Solutions | Marks | Remarks |
|---|---|---|---|
| 1. | $a^2 - a - 6 = (a + 2)(a - 3)$ ) ) ...... any 1 all correct | 2A+1A | 2A for first correct part |
| | $a^3 + 8 = (a + 2)(a^2 - 2a + 4)$ ) | | |
| | Their L.C.M. $= (a + 2)(a - 3)(a^2 - 2a + 4)$ the other correct | 1M+1A | Both exp. must first be factorized. |
| | $(= a^4 - 3a^2 + 8a - 24)$ | | |
| | | 5 | PP-1 at most 3 per paper, at most 1 per question, at most 1 for the same type of pp. |
| 2. | (a) $\dfrac{\sin(180° - \theta)}{\sin(90° + \theta)} = \dfrac{\sin\theta}{\cos\theta}$ must be shown.......... | 1A<br>1A | EXCESS |
| | $= \tan\theta$ | 1A | |
| | (b) $\sin^2(\pi - \phi) + \sin^2(\frac{3\pi}{2} + \phi)$ | | |
| | $= \sin^2\phi + \cos^2\phi$ $\sin^2\phi - \cos^2\phi$ ... 0A | 1A | For $\sin(\frac{3\pi}{2} + \phi) = -\cos\phi$ |
| | $= 1$ .................................... | 1A | |
| | | 5 | |
| 3. | $2x^2 \geqslant 5x$ | | With-hold 1 mark if '=' omitted. If solved by equation, no marks awarded unless answer correct. |
| | $2x^2 - 5x \geqslant 0$ | 1A | |
| | $x(2x - 5) \geqslant 0$ .................................. | 1A | |
| | Case (i) $x \geqslant 0$ and $2x - 5 \geqslant 0$ | | Optional Any 1 part without '=', withhold 1 mark. |
| | i.e. $x \geqslant \frac{5}{2}$ | | |
| | Case (ii) $x \leqslant 0$ and $2x - 5 \leqslant 0$ | | |
| | i.e. $x \leqslant 0$ | | |
| | Combining the two parts, we have $x \leqslant 0$ or $x \geqslant \frac{5}{2}$. | 3A | For $x \leqslant 0$, $x \geqslant \frac{5}{2}$, 2 |
| | | | $x \leqslant 0$ and $x \geqslant \frac{5}{2}$ 1 |
| | | 5 | |
| 4. | (a) If $9x^2 - (k + 1)x + 1 = 0$ has equal roots, | | **Alt. Solution:** |
| | $(k + 1)^2 - 36 = 0$ .......................... | 1A | $(k+1)^2 - 36 = 0$ 1A |
| | $k^2 + 2k - 35 = 0$ | 1A | $k + 1 = \pm 6$ 1A+1A |
| | $(k - 5)(k + 7) = 0$ | 1A | $k = 5$ or $-7$ 1A |
| | $k = 5$ or $-7$ both correct | 1A | $k+1 = 6$ 1A only |
| | (b) Putting $k = -7$ in (*) | 1M | For Sub. negative value of k |
| | $9x^2 + 6x + 1 = 0$ | | |
| | $(3x + 1)^2 = 0$ | | L.S. $= (3x + 1)^2$ |
| | $x = -\frac{1}{3}$ subs. both for k=-7 and k=5 no marks | 1A | $x = -\frac{1}{3}$ |
| | | 6 | |

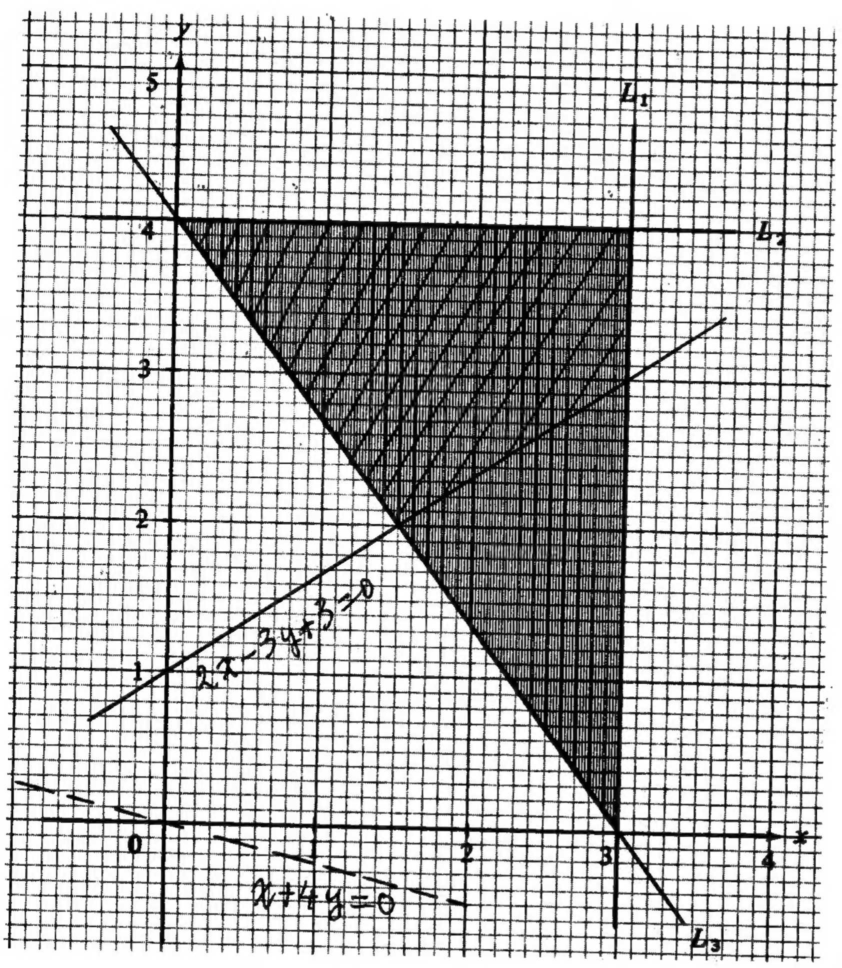
| Solutions | Marks | Remarks |
|---|---|---|
| 5. (a) Area of OABC $= \pi 10^2 \times \frac{100°}{360°}$ ................ | 1M | |
| $= 87.27$ (corr. to 2 d.p.) (or 87.28) | 1A | |
| (b) Area of $\triangle$ OAC $= \frac{1}{2} \times 10 \times 10 \times \sin 100°$ | 1M | $\triangle = \frac{1}{2} AC \times OM$ |
| $= 49.24$ (corr. to 2 d.p.) | 1A | $= \frac{1}{2} \times 15.3209 \times 6.4279$ ... 1M |
| | | $= 49.24$ ... 1A |
| (c) Area of minor segment ABC | |   |
| $= 87.27 - 49.24$ ............................ | 1M | |
| $= 38.03$ (corr. to 2 d.p.) (or 38.04) | 1A | |
| | 6 | |
| 6. $\log 2 = r$, $\log 3 = s$. | | |
| (a) $\log 18 = \log 2 \times 3^2$ | 1A | For $18 = 2 \times 3^2$ |
| $= \log 2 + \log 3^2$ ) ....................<br>$= \log 2 + 2\log 3$ ) | 1M | ) $\log ab = \log a + \log b$ or<br>) $\log a^2 = 2\log a$ |
| $= r + 2s$ ............................ | 1A | |
| (b) $\log 15 = \log 3 \times 5$ | | |
| $= \log 3 + \log 5$ 1A | | |
| $= \log 3 + \log \frac{10}{2}$ .................... | 1A | For $5 = \frac{10}{2}$ or $15 = \frac{30}{2}$ |
| $= \log 3 + \log 10 - \log 2$ | 1A | |
| $= 1 - r + s$ ........................ | 1A | |
| | 6 | |
| 7. (a) The coordinates of the centre are given by | | |
| $x = -(-\frac{4}{2})$, $y = -\frac{10}{2}$ .................... | 1M | $(x-2)^2 + (y+5)^2 = 25$ $k+4$ |
| only answer correct 2A → i.e. $x = 2$, $y = -5$   2,-5 without bracket PP-1 | 1A | |
| (b) As C touches the y-axis, | | OR |
| its radius = 2 ............................ | 1M+1A | Subs. (0, −5) 1M<br>$25 - 50 + k = 0$<br>$k = 25$ 1A |
| $4 + 25 - k = 2^2$ | 1M | $r = \sqrt{4 + 25 - 25}$ 1M<br>$= 2$ .......... 1A |
| $k = 25$ | 1A | OR |
| | | Put $x = 0$,<br>$y^2 + 10y + k = 0$<br>has equal roots. 1M<br>$100 - 4k = 0$<br>$k = 25$ 1A<br>$r =$ etc. |
| | 6 | |

| Solutions | Marks | Remarks |
|---|---|---|
| 8. (a) (i)  | | ABCD in must order |
| | 1 | For P |
| | 1 | For Q (between D, C) |
| (ii) Since $\Delta$ PBC is equilateral, $\angle$ PBC = 60° | | Follow through even if diagram not accurate |
| angles written down on the diagram → no method mark<br>$\angle$ABP = 90° − 60° = 30° ............. | 1A | |
| As BA = BP , $\angle$ PAB = $\frac{1}{2}$ (180° − 30°) | 1M | or equivalent |
| = 75° | 1A | OR |
| Since AB // DC, $\angle$ PQC = 180° − 75° | 1M | $\angle$PAD = 15° |
| = 105° | 1A | $\angle$ PQC = 90° + 15° 1M<br>= 105° 1A |
| | 7 | |
| (b) (i) $\Delta$ TCB is similar to $\Delta$ ACT because | 1 | |
| both correct 1 mark { $\angle$ C is common. | | ≈ ≅ } PP-1 |
| $\angle$ BTC = $\angle$ BAT (angle no need in alternate segment)<br>$\angle$T no mark | 1 | Indication of 2 pairs of equal angles. Withheld if proving congruence. |
| △TCB ~ △ACT (~~A.A.A.~~) → no mark<br>1 mark | | |
| (ii) $\frac{AC}{CT} = \frac{CT}{BC}$ ............................. | 1A | Follow through even if (b)(i) wrong. |
| AC = $\frac{6^2}{5}$ = 7.2 correct substitution | 1A | |
| $\therefore$ AB = 7.2 − 5 | | |
| = 2.2 ( = $\frac{11}{5}$ ) ............... | 1A | |
| | 5 | |
|  | | |

| Solutions | Marks | Remarks |
|---|---|---|
| 9. (a) Between 100 and 999, | | |
| the smallest multiple of 7 is 105, | 1A | |
| the largest is 994. | 1A | |
| | 2 | |
| (b) The number of multiples is $\frac{994 - 105}{7} + 1$ must be correct. | 2M | OR 994= 105 + (n-1) X 7 |
| $= 128$ | 1A | |
| The sum of these multiples | | |
| $= 105 + 112 + \ldots + 994$ | | |
| $= \frac{128}{2}[105 + 994]$ ......................... | 2M | |
| $= 70336$ | 1A | |
| | 6 | |
| (c) The sum of all positive 3-digit integers | 1 | |
| $= 100 + 101 + \ldots + 999$ or all correct | ~~1A~~ | |
| $= \frac{900}{2}[100 + 999]$ | | |
| $= 494{,}550$ ................................ | 1A | |
| The required sum $= 494{,}550 - 70{,}336$ | 1M | |
| $= 424{,}214$ | 1A | |
| | 4 | |

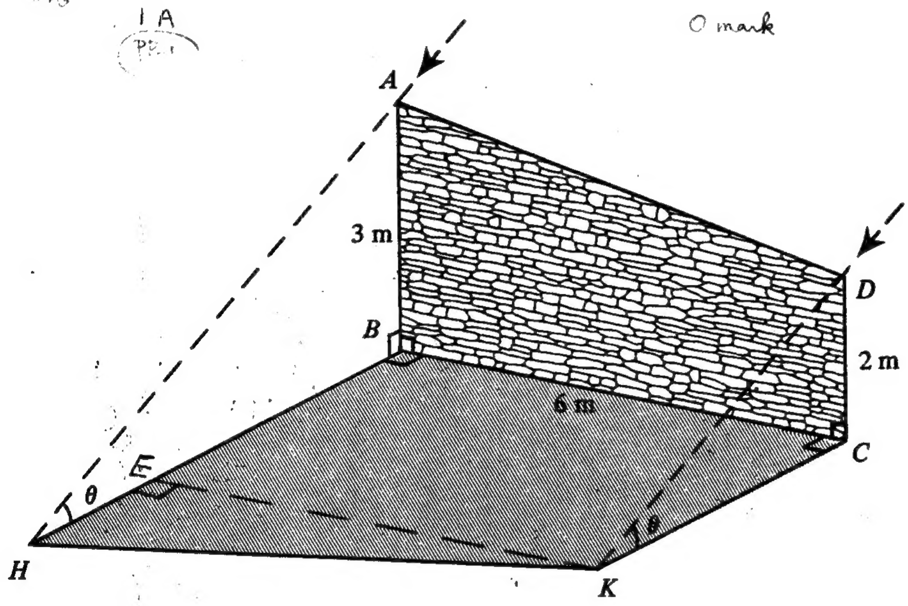
| Solutions | Marks | Remarks |
|---|---|---|
| 10. (a) Let $y = k_1x + k_2x^2$, where $k_1$ and $k_2$ are constants. | | For $y=kx+kx^2$ or $y = kx+x^2$ or $y = x+kx^2$ ....... 1 |
| for substitution | 2 | |
| Putting $x = 1$, $y = -5$; $x = 2$, $y = -8$, we have | 1M | $y = x + x^2$ no marks |
| $k_1 + k_2 = -5$ ....................... | 1A | no marks $\begin{cases} y = k_1x \\ y = k_2x^2 \end{cases}$ |
| $2k_1 + 4k_2 = -8$ | 1A | |
| Solving, $k_1 = -6$, $k_2 = 1$ | 1A+1A | |
| $\therefore$ $y = -6x + x^2$ | | |
| Putting $x = 6$, we have $y = 0$. | 1A | |
| | 8 | |
| (b) $y = -6x + x^2 = (x^2 - 6x + 9) - 9$ | 1M | Equality must hold. |
| $= (x - 3)^2 - 9$ ................ | 1A | $y = (x+3)^2 - 9$ 0A |
| When $x = 3$, the value of $y$ is least and the least value is $-9$. | 1M+1A | least value of $y$ is $-9$ 1M 0A |
| | 4 | |
| 11. (a) From the curve, | | |
| (i) the median is 70 marks. | 1A | |
| (ii) the 1st quartile is 50 marks. ) the 3rd quartile is 86 marks. ) ...... | 1A | for either |
| $\therefore$ the interquartile range $= 86 - 50$ | 1M | |
| $= 36$ marks | 1A | |
| | 4 | |
| (b) (i) From the curve, the number of prize-winners = 60. | 1A | |
| (ii) The probability that the student is a prize-winner $= \frac{60}{600}$ 1M $\left( = \frac{1}{10} \right)$. | 1M+1A | |
| (iii)(1) The probability that both are prize-winners is $\frac{60}{600}$ 1M $\times \frac{59}{599} = \frac{59}{5990}$ 1A $(=0.01)$ | 1M+1A | Accept $\frac{1}{10} \times \frac{1}{10} = \frac{1}{100}$ 1M for product rule |
| (2) The probability that both are not prize-winners $= \frac{540}{600}$ 1A $\times \frac{539}{599}$ $\left(= \frac{4851}{5990}\right)$ $(=0.81)$ | 1A | Accept $\frac{9}{10} \times \frac{9}{10}$ |
| $\therefore$ the probability that at least one is a prize-winner $= 1 -$ 1M $\frac{4851}{5990}$ | 1M | **OR** $\frac{9}{10} \times \frac{60}{599} + \frac{1}{10} \times \frac{540}{599}$ |
| 1A $= \frac{1139}{5990}$ $(=0.19)$ | 1A | $+ \frac{1}{10} \times \frac{59}{599}$ 1M+1A |
| | | $= \frac{1139}{5990}$ ....... 1A |
| | 8 | |

| Solutions | Marks | Remarks |
|---|---|---|
| 12. (a) $L_3$ is given by $\frac{x}{3} + \frac{y}{4} = 1$    $\frac{y-4}{x} = \frac{4}{3}$ wrong slope | 1M | or 2-pt form, etc. |
| i.e. $4x + 3y = 12$ ..............................   1M   $\frac{x}{4} + \frac{y}{3} = 1$ | 1A<br>2 | Must be in this form. |
| (b) The three constraints are $y \leqslant 4$ | 1A | Withhold 1 mark if '=' omitted. |
| $x \leqslant 3$ | 1A | |
| $4x + 3y \geqslant 12$ .......... | 1A<br>3 | or $4x + 3y - 12 \geqslant 0$. |
| (c) The line $x + 4y = c$ drawn in the diagram. | 1M+1A | For 1A<br>Drop of 2-3 verticle units for 10 hori-zontal units.<br>OR Testing any vertices ...... 1M |
| From the diagram, P is greatest when $x = 3$, $y = 4$ and least when $x = 3$, $y = 0$. | | |
| only answer 2A { The greatest value of $P = 19$, | 1A | At $(3, 0)$, $P = 3$.<br>At $(0, 4)$, $P = 16$. |
| the least value = 3. ...................... | 1A<br>4 | At $(3, 4)$, $P = 19$.   1A<br>test 2 points only 1M |




| Solutions | Marks | Remarks |
|---|---|---|
| (d) The line $2x - 3y + 3 = 0$ drawn in the diagram. | 1A | ±1 unit at $(1.5, 2)$, $(3, 3)$. |
| The shaded region. | 1A | Should be reasonably shaded. |
| P is least when $x = \frac{3}{2}$, $y = 2$. | | At $(3, 3)$, $P = 15$.<br>At $(1.5, 2)$, $P = 9.5$. |
| The least value $= \frac{19}{2}$ $(= 9.5)$ ............... | 1A<br>3 | |

| Solutions | Marks | Remarks |
|---|---|---|
| 13. (a) $\frac{AB}{HB} = \tan\theta$ | | any part in this question Wrong/no unit, pp-1. in the answer in each part |
| $HB = \frac{3}{\tan\theta}$ m .............................. | 1M<br>1A | 2 + 1 |
| 1M or $\frac{DC}{KC} = \tan\theta$, $KC = \frac{2}{\tan\theta}$ m | 1A | |
| | 3 | |
| (b) (i) $S_1 = \frac{6}{2}(3 + 2)$ | | |
| $= 15\ m^2$ .............................. | 1A | |
| (ii) $S_2 = \frac{6}{2}\left(\frac{3}{\tan\theta} + \frac{2}{\tan\theta}\right)$ | | |
| $= \frac{15}{\tan\theta}\ m^2$ .............................. | 1A | |
| $\therefore \frac{S_1}{S_2} = \frac{15}{\frac{15}{\tan\theta}} = \tan\theta$ | 1A | Must show working. |
| $\frac{15}{(\frac{15}{\tan\theta})} = \tan\theta$ ; $\tan\theta = \tan\theta$ 0 mark | | $\frac{15}{\frac{15}{\tan\theta}}$ $= \tan\theta$ 1A (PP-1) ; $\frac{15}{\frac{15}{\tan\theta}}$ $= \tan\theta$ 1A (PP-1) |
| $\frac{15}{\frac{15}{\tan\theta}} = \tan\theta$ 1A (PP-1) | 3 | |




| Solutions | Marks | Remarks |
|---|---|---|
| (c) Let $KE \perp BH$. | 1M | Construction of perpendicular line |
| $EK = BC = 6$ (m)    K? = 6 —— 2 marks | 1A | |
| $HE = \frac{3}{\tan\theta} \overset{1M}{-} \frac{2}{\tan\theta} = \left(\frac{3}{\tan 30^\circ} - \frac{2}{\tan 30^\circ}\right)$ m $(= \sqrt{3})$   1M $\theta = 30^\circ$ | 1M+1M | HB=5.1961... or 5.2<br>KC=3.464... or 3.5<br>HE=1.732<br>HK=6.24 |
| $\therefore HK = \sqrt{HE^2 + EK^2}$ | 1M | |
| $= \sqrt{(\sqrt{3})^2 + 6^2}$ | | |
| $= \sqrt{39}$ m .............................. | 1A | |
| | 6 | |

| Solutions | Marks | Remarks |
|---|---|---|
| 14. (a) (i) $x^3 - \frac{4}{3}x - 6 = 0$ can be written as | | |
| $x^3 = \frac{4}{3}x + 6$ . | 1M | |
| Consider the line $y = \frac{4}{3}x + 6$ | 1A+1A | 1A for equation<br>1A for line drawn,<br>±1 vertical division<br>about (0, 6), (3, 10) |
| It cuts the curve $y = x^3$ at $x = r$ , | | |
| where $r$ lies between 2.0 and 2.1 . | 1A | |
| (ii) Let $f(x) = x^3 - \frac{4}{3}x - 6$ | | |
| 無 but $f(2) = -(= -0.67)$<br>$f(2.1) = +(=0.46)$ both correct ........................ | 1M | Correct change of sign. |

| Interval | Mid-value x | f(x) |
|---|---|---|
| $2.000 < r < 2.100$ | 2.050 1M | 1A $-(=-0.12)$ |
| 1M $2.050 < r < 2.100$ | 2.075 | $+(=0.17)$ |
| $2.050 < r < 2.075$ | 2.063 | $+(=0.02)$ |
| $2.050 < r < 2.063$ | 2.057 | $-(=-0.04)$ |
| $2.057 < r < 2.063$ | | |

all correct give 5 marks

Marks: 1M+1A, 1M — Remarks: 1M for choosing mid-value, 1A for correct sign. Next correct interval step.

$\therefore$ $r = 2.06$ (correct to 2 d.p.) — 1A

Total: 9

**Alt. Solution:**

$f(2) = -$
$f(2.5) = +$ ) ........ — 1M

2.25 0M+0A

| Interval | Mid-value x | f(x) |
|---|---|---|
| $2.000 < r < 2.500$ | 2.250 | + |
| $2.000 < r < 2.225$ | 2.113 | + |
| ⋮ | ⋮ | ⋮ |

Marks: 1M+1A, 1M

$\therefore$ $r = 2.06$ (correct to 2 d.p.) — 1A

(b) Put $x = t + 1$ — 1A

The given equation can be written

as $3x^3 - 4x - 18 = 0$

or $x^3 - \frac{4}{3}x - 6 = 0$

By (a), the solution is

$t = 2.06 - 1$ ......................... — 1M

$= 1.06$ (correct to 2 d.p.) — 1A

Total: 3

| Solutions | Marks | Remarks |
|---|---|---|

14.